RATIO-TYPE ESTIMATORS IN DOUBLE SAMPLING FOR TWO-STAGE DESIGNS*

By

R.C. Garg* and S.S. Pillai

Institute of Agricultural Research Statistics, New Delhi.

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1. Introduction

One of the significant developments in the theory of sampling in the last two decades is the use of ancillary information for obtaining estimates of the population characteristics with improved precision. When such information is available, various estimators for the population mean or total have been obtained for different sampling designs by using different methods of estimation. When the design adopted is simple random sampling without replacement, it is known that the estimates built by ratio method of estimation are biased. This bias is not always negligible. Thus, various research workers have estimated the extent of bias and thereby formulated unbiased ratio-type estimators.

For a single phase design, Hartley and Ross (1954) developed one ratio-type estimate for obtaining the unbiased estimate of the population mean. Robson (1957) derived the exact formula for the variance of this estimate. When the finite population correction factor is negligible, Goodman and Hartley (1958) obtained the variance of this estimated mean along with an unbiased estimator for the variance.

For a single-stage two-phase design, Sukhatme (1962) got the unbiased ratio-type estimator and its variance using the technique developed by Tukey (1956) and further extended by Robson (1957).

For a two-stage single phase design, Ross (1960) obtained the unbiased ratio-type estimate and its variance and also discussed its efficiency.

In this paper, two ratio-type estimates of the population mean in the case of two-stage sampling have been presented when the ancillary information is not available for all the units in the population.

* Present address: Junior Programmer, Indian Veterinary Research Institute, Izatnagar, U.P.

Using simple cost functions, the optimum allocation of the sample units for attaining a given precision, the total cost of the survey being fixed, have been worked out when two-phase sampling scheme in two stages is adopted.

Finally, for gètting the unbiased estimate and the variance of biased and unbiased estimates, the same technique of symmetric means developed by Tukey and extended by Robson (1957) has been adopted for a sub-sampling design. A brief account of the technique [vide Robson (1957)] together with the multiplication formula for the product of two symmetric means is given below :—

The polynomial

$$\frac{1}{(n)_r}\sum_{j_1\neq j_2\neq \ldots \neq J_r}\left(x_{1j1}^{a_{11}}\quad x_{2j1}^{a_{21}}\ \ldots\ \ x_{mj1}^{a_{m1}}\right)\ldots\left(x_{1jr}^{a_{1r}}\quad x_{2jr}^{a_{2r}}\ \ldots\ \ x_{mjr}^{a_{mr}}\right)$$

in the mn variates x_{ij}, $i=1, \ldots m$; $J=i, \ldots n$, is called a symmetric mean and is denoted by

$$<(\alpha_r)>=<(a_1{}^*)\ (a_2{}^*)\ \ldots\ (a_r{}^*)>$$

where (a_1^*) is the vector $(a_{1i}, \ldots a_{mi})$.

Consider now two symmetric means

$$<(\alpha_r)>=<(a_1{}^*)\ (a_2{}^*)\ \ldots\ (a_r{}^*)> \text{ and}$$
$$<(\beta_s)>=<(b_1{}^*)\ (b_2{}^*)\ \ldots\ (b_s{}^*)>$$

Let

$$P_y(\alpha_r\beta_s)=[(a_{i1}+b_{j1})\ \ldots\ (a_{ir}+b_{jy}),$$
$$(a_{ir}+1)\ \ldots\ (a_{ir}),\ (b_{jy}+1)\ \ldots\ (b_{js})]$$

be obtained by pairing and adding Y elements of (α_r) with Y elements of (β_s). Let us denote by $Ry\ (\alpha_r\beta_s)=[Py(\alpha_r\beta_s)]$ the set of all possible $Y_i \binom{r}{y}\binom{s}{y}$ possible sets $P_y(\alpha_r\beta_s)$. Then the product of two symmetric means $<(\alpha_r)>$ and $<(\beta_s)>$ is given by the formula

$$<(\alpha_r)><(\beta_s)>=\frac{1}{(n)_r(n)_s}\sum_{Y=0}^{r}(n)_{r+s-y}\sum_{R_y}<P_y(\alpha_r\beta_s),>,\ r\leqslant s$$

It can be seen that if these mn variates represent a simple random samble of n observations from an m dimensional finite population of size N, the expected value of the Statistics $<(\alpha_r)>$ taken over $\binom{N}{n}$ possible samples is the corresponding symmetric mean of the population.

2. RATIO-TYPE ESTIMATORS FOR A TWO-STAGE DESIGN WHEN THE POPULATION MEAN OF THE AUXILIARY CHARACTER IS NOT KNOWN

In a situation where the auxiliary information is not available for all the units in the population, the procedure of double sampling is usually adopted. In this procedure, a preliminary sample of size 'n'' is selected from N first-stage units and from each selected first-stage unit consisting of M second-stage units. (Equality of M is applicable for large values of M). 'm'' units are drawn for observing X, the ancillary character. Again a random sub-sample of size n is drawn from n' and from each first-stage unit thus selected, m units are drawn at random to observe Y, the character under study. Using this sampling scheme, we shall consider the problem of estimating the population mean. Following notations developed by Tukey, for a three-variate population

$$(x_{ij}, y_{ij}, r_{ij}), j=1, 2 \ldots M \text{ and } i=1, 2 \ldots N,$$

where

$$r_{ij}=y_{ij}/x_{ij},$$

we have

$$\bar{Y}_{NM}=<010>''=<101>''$$

=the mean per element of Y in the population

$$\bar{X}_{NM}=<100>''$$

=the mean per element of X in the population

$$\bar{R}_{NM}=<001>''$$

=the mean per element of the ratio of Y to X in the population

$$\bar{Y}_{i\cdot}=<010>'i=<101>'i$$

=the mean per element of Y in the ith primary unit in the population

$$\bar{X}_{i\cdot}=<100>'i$$

=the mean per element of X in the ith primary unit in the population

$$\bar{R}_{i\cdot}=<001>'i$$

=the mean per element of the ratio of Y to X in the ith primary unit in the population

$$\bar{y}_{nm}=\frac{1}{n}\sum_{i}^{n} <010>_i$$

= the mean per element of Y in the sample

$$\bar{x}_{nm} = \frac{1}{n}\sum_{i}^{n} \langle 100\rangle_i$$

= the mean per element of X in the sample

$$\bar{x}_{n'm'} = \frac{1}{n'}\sum_{i}^{n'} \langle 100\rangle_i^*$$

= the mean per element of X in the large sample

$$\bar{r}_{nm} = \frac{1}{n}\sum_{i}^{n} \langle 001\rangle_i$$

= the mean per element of the ratio of Y to X in the sample.

$$\bar{y}_{im} = \langle 010\rangle_i$$

= the mean per element of Y in the ith primary unit in the sample

$$\bar{x}_{im} = \langle 100\rangle_i$$

= the mean per element of X in the ith primary unit in the sample

$$\bar{x}_{im'} = \langle 100\rangle_i^*$$

= the mean per element of X in the ith primary unit in the large sample

$$\bar{r}_{im} = \langle 001\rangle_i$$

= the mean per element of the ratio of Y to X in the ith primary unit in the sample

We shall now consider the following ratio-type estimate :

$$T_1 = \bar{r}_{nm}\bar{x}_{n'm'} \qquad \text{...(2.1)}$$

It can be easily verified that T_1 is biased. An unbiased estimate of the bias in T_1 can be shown to be equal to

$$(\bar{y}_{nm} - \bar{r}_{nm}\bar{x}_{nm}) + \left(\frac{1}{n} - \frac{1}{n'}\right) s_{brx}$$

$$+ \frac{1}{n'}\left(\frac{1}{m} - \frac{1}{m'}\right)\frac{1}{n} \sum^{n} s_{irx}$$

where

$$s_{brx}=\frac{1}{n-1}\sum_{i}^{n}(\bar{r}_{im}-\bar{r}_{nm})(\bar{x}_{im}-\bar{x}_{nm})$$

and

$$s_{irx}=\frac{1}{m-1}\sum_{j}^{m}(r_{ij}-\bar{r}_{im})(x_{ij}-\bar{x}_{im})$$

Correcting T_1 for bias, an unbiased estimate for $\bar{Y}_{NM}$ will be

$$T_2=T_1+(\bar{y}_{nm}-\bar{r}_{nm}\bar{x}_{nm})+\left(\frac{1}{n}-\frac{1}{n'}\right)s_{brx}$$

$$+\frac{1}{n'}\left(\frac{1}{m}-\frac{1}{m'}\right)\frac{1}{n}\sum^{n}s_{irx} \quad \ldots (2.2)$$

Using the concept of symmetric means, the variance estimators of T_1 and T_2 can be derived. As an illustration, the variance of T_1 has been derived as follows :

$$V(T_1)=E_1[E_2(T_1^2)/n']-[E_1E_2(T_1/n')]^2 \quad \ldots (2.3)$$

where the symbols V, E_1, E_2 have their usual meaning. (by definition)

Considering the first Component of R.H.S. in (2.3),

$$E_2(T_1^2)/n'=E_2\left[\frac{1}{n}\sum_{i}^{n}<001>_i\frac{1}{n}\sum_{i}^{n}<001>_i\right]\left[\frac{1}{n'}\sum^{n'}<100>_i^*\right]^2$$

The expected value of a sample symmetric mean being the corresponding symmetric mean evaluated over all the units in the population, the expected value given above can be simplified only if we express it as a linear function of symmetric means. Using the multiplication formula for the product of two symmetric means, we have

$$E_2(T_1^2/n')=\left[\frac{1}{nm}\frac{1}{n'}\sum_{i}^{n'}<002>_i^*+\frac{m-1}{nm}\frac{1}{n'}\sum_{i}^{n'}<(001)(001)>_i^*\right.$$

$$\left.+\frac{n-1}{nn'(n'-1)}\sum_{i\neq i'}^{n'}<001>_i^*<001>_{i'}^*\right]\times$$

$$\left[\frac{1}{n'^2}\sum_{i}^{n'}<100>_{i'}^*<100>_i^*+\frac{1}{n'^2}\sum_{i\neq i'}^{n'}<100>_i^*<100>_{i'}^*\right]$$

Applying multiplication rule again, taking expectation and retaining terms of the order of $\frac{1}{nm}$ or $\frac{1}{n'm'}$,

We get

$$E_1(E_2(T_1^2/n')) = \frac{(n-1)(n'-2)}{nn'^2m\; N(N-1)(N-2)}$$

$$\Big[\sum_{i\neq i'\neq i''}^{N} \langle 001\rangle'_i\, \langle 001\rangle'_{i'}\, \langle 200\rangle'_{i''} + (m'-1)\sum_{i\neq i'\neq i''}^{N} \langle 001\rangle'_i \; \langle 001\rangle'_{i'}\, \langle (100)(100)\rangle'_{i''}\Big]$$

$$+\frac{(n'-1)(n'-2)}{nm\,n'^2 N(N-1)(N-2)} \sum_{i\neq i'\neq i''}^{N} \langle 100\rangle'_i\, \langle 100\rangle'_{i'}\, \langle 002\rangle'_{i''}$$

$$+\frac{(m-1)(n'-1)(n'-2)}{nmn'^2 N(N-1)(N-2)} \sum_{i\neq i'\neq i''}^{N} \langle 100\rangle'_i\, \langle 100\rangle'_{i'}\, \langle (001)(001)\rangle'_{i''}$$

$$+\frac{4(n-1)(n'-2)}{nm'n'^2 N(N-1)(N-2)} \Big[\sum_{i\neq i'\neq i''}^{N} \langle 001\rangle'_{i'}\, \langle 100\rangle'_{i''}\, \langle 101\rangle'_i$$

$$+ (m'-1) \sum_{i\neq i'\neq i''}^{N} \langle 001\rangle'_{i'}\, \langle 100\rangle'_{i''}\, \langle (100)(001)\rangle'_i\Big]$$

$$+\frac{(n-1)(n'-2)(n'-3)}{nn'^2 N(N-1)(N-2)(N-3)} \sum_{i\neq i'\neq i''\neq i'''}^{N} \langle 001\rangle'_i\, \langle 001\rangle'_{i'}\, \langle 100\rangle'_{i''}\, \langle 100\rangle'_{i'''} \quad \ldots (2.4)$$

Similarly

$$\Big[E_1E_2(T_1/n')\Big]^2 = \frac{(n'-1)^2}{n'^2}\Big[\frac{1}{N(N-1)} \sum_{i\neq i'}^{N} \langle 001\rangle'_i\, \langle 100\rangle'_{i'}\Big]^2$$

$$+\frac{2(n'-1)}{n'^2m'\,N(N-1)} \langle 010\rangle' \sum_{i\neq i'}^{N} \langle 001\rangle'_i\, \langle 100\rangle'_{i'}$$

$$+\frac{2(n'-1)(m'-1)}{n'^2m'\,N(N-1)} \langle (100)(001)\rangle'' \sum_{i\neq i'}^{N} \langle 001\rangle'_i\, \langle 100\rangle'_{i'} \quad \ldots (2.5)$$

Substituting (2.4) and (2.5) in (2.3) and converting back the symmetric means into standard notations, we have

$$V(T_1)=V(\bar{r}_{nm})\bar{X}_{nm}^2+V(\bar{X}_{n'm'})\bar{R}_{NM}^{\ 2}$$
$$+2\bar{R}_{NM}\bar{X}_{NM}\ \text{Cov.}\ (\bar{r}_{nm},\ \bar{x}_{n'm'}). \qquad \text{... (2.6)}$$

Similarly

$$V(T_2)=V(\bar{y}_{nm})+V(\bar{x}_{nm})\ \bar{R}_{NM}^2-V(\bar{x}_{n'm'})\bar{R}_{NM}^2\ \text{Cov.}\ (\bar{y}_{nm},\bar{x}_{nm})R_{NM}$$
$$-2\ \text{Cov.}\ (\bar{y}_{nm},\ \bar{x}_{n'm'})\ \bar{R}_{NM} \qquad \text{... (2.7)}$$

Mean Square Error (M.S.E.) of T_1 is given by

$$V(T_1)+(\text{Bias})^2$$

$$MSE\ (T_1)=V(\bar{r}_{nm})\bar{X}_{NM}^2+V(\bar{x}_{n'm'})\bar{R}_{NM}^2$$
$$+4\ \text{Cov.}\ (\bar{r}_{nm},\ \bar{x}_{n'm'})\bar{R}_{NM}\bar{X}_{NM}$$
$$+\text{Cov.}\ (\bar{r}_{nm},\ \bar{x}_{n'm'})^2+(\bar{R}_{NM}\bar{X}_{NM}-\bar{Y}_{NM})^2$$
$$-2\ \bar{Y}_{NM}\ \text{Cov.}\ (\bar{r}_{nm},\ \bar{x}_{n'm'}) \qquad \text{... (2.8)}$$

We shall now investigate the efficiency of the unbiased estimator T_2 as compared to the estimator T_1. Comparing the mean square errors of T_1 and T_2, it can be shown that T_2 will be more efficient than T_1 if the following conditions are satisfied :

$$P_{bxy}>\frac{1}{2}\frac{S^2_{by}+S^2_{bx}\bar{R}_{NM}^2-S^2_{br}\bar{X}_{NM}^2}{S_{bx}S_{by}} \qquad \text{... (2.9)}$$

$$\bar{P}_{wxy}>\frac{1}{2}\frac{\bar{S}^2_{wy}+\bar{S}_{wx}^2\bar{R}_{NM}^2-\bar{S}_{wr}^2\bar{X}_{NM}^2}{\bar{S}_{wx}\bar{S}_{wy}} \qquad \text{... (2.10)}$$

$$P_{brx}>\frac{\bar{R}_{NM}}{S_{br}}\left[\frac{P_{bxy}S_{by}-S_{br}\bar{R}_{NM}}{2\ \bar{R}_{NM}\bar{X}_{NM}-\bar{Y}_{NM}}\right] \qquad \text{... (2.11)}$$

and

$$\bar{P}_{wrx}>\frac{\bar{R}_{NM}}{\bar{S}_{wr}}\left[\frac{\bar{P}_{wxy}\bar{S}_{wy}-\bar{S}_{wx}\bar{R}NM}{2.\bar{R}_{NM}.\bar{X}_{NM}-\bar{Y}NM}\right] \qquad \text{... (2.12)}$$

where

$$P_{bxy}=\frac{S_{bxy}}{S_{bx}\ S_{by}}$$

$$P_{brx}=\frac{S_{brx}}{S_{br}\ S_{bx}}$$

$$\overline{P}_{wxy}=\frac{\overline{S}_{wxy}}{\overline{S}_{wx}\,\overline{S}_{wy}}=\frac{\frac{1}{N}\sum_{i}^{N} S_{ixy}}{\left(\frac{1}{N}\sum_{i}^{N}S^2_{ix}\cdot\frac{1}{N}\sum_{i}^{N}S^2_{iy}\right)^{\frac{1}{2}}}$$

Similarly

$$\overline{P}_{wrx}=\frac{\overline{S}_{wrx}}{\overline{S}_{wr}\,\overline{S}_{wx}}$$

3. OPTIMUM ALLOCATION OF THE SAMPLE FOR THE UNBIASED ESTIMATOR T_2

In this section we shall consider the problem of optimum allocation of the sample units when the double sampling is adopted in a two-stage design. Let C_0 denote the total cost of the survey which may, in this case, be expressed as,

$$C_0=C_1n+C_2nm+C_3n'+C_4n'm' \qquad \text{... (3.1)}$$

where C_1=the cost of visiting a primary unit in the second phase sample.

C_2=the cost per second-stage unit in making observations and recording the same.

C_3=the cost of visiting a primary unit in the first phase sample.

and C_4=the cost per second-stage unit in making and recording the observation on character X.

Then the optimum variance of estimator T_2 for large N and M and for first order approximation, is given by

$$V(T_2)\text{ opt.}=\frac{(\sqrt{C_1V_1}+\sqrt{C_2V_2}+\sqrt{C_3V_3}+\sqrt{C_4V_4})^2}{C_0}$$

where

$$V_1=S^2_{by}+S^2_{bx}\,\overline{R}^2_{NM}-2\,\overline{R}_{NM}\,S_{bxy} \qquad \text{... (3.2)}$$

$$V_2=\frac{1}{N}\sum_{i=1}^{N} S^2_{iy}+\frac{1}{N}\sum_{i=1}^{N} S^2_{ix}\,\overline{R}^2_{NM}-2\,\overline{R}_{NM}\frac{1}{N}\sum_{i=1}^{N} S_{ixy} \qquad \text{.. (3.3)}$$

$$V_3=2\,\overline{R}_{NM}\,S_{bxy}-S^2_{bx}\,\overline{R}^2_{NM} \qquad \text{... (3.4)}$$

$$V_4=\frac{2}{N}\sum_{i=1}^{N} S_{ixy}\,\overline{R}_{NM}-\frac{1}{N}\sum_{i=1}^{N} S^2_{ix}\,\overline{R}^2_{NM} \qquad \text{... (3.5)}$$

4. Estimates of Variance

Consistent estimates of variance of T_1 and T_2 can be obtained directly by replacing the population values by their corresponding sample values. These are given by

$$\hat{V}(T_1)=\hat{V}(\bar{r}_{nm})\bar{x}^2_{nm}+\hat{V}(\bar{x}_{n'm'})\,\bar{r}^2_{nm}+2\,\bar{r}_{nm}\bar{x}_{nm}\,\hat{\text{Cov}}.\,(\bar{r}_{nm},\,\bar{x}_{n'm'}) \quad \ldots (4.1)$$

$$\hat{V}(T_2)=\hat{V}(\bar{y}_{nm})+\hat{V}(\bar{x}_{nm})\,\bar{r}^2_{nm}+2\,\hat{\text{Cov}}.\,(\bar{y}_{nm},\,\bar{x}_{n'm'})\,\bar{r}_{nm} -\hat{V}(\bar{x}_{n'm'})\,\bar{r}^2_{nm}-2\,\bar{r}_{nm}\,\hat{\text{COV}}\,(\bar{y}_{nm},\,\bar{x}_{nm}) \quad \ldots (4.2)$$

where

$$\hat{V}(\bar{r}_{nm})=\frac{1}{n(n-1)}\left[\sum_{i}^{n}\bar{r}_{im}-n\;\bar{r}^2_{nm}\right]$$

$$\hat{V}(\bar{x}_{n'm'})=\frac{1}{n'(n-1)}\left[\sum_{i}^{n'}\bar{x}^2_{im'}-n'\bar{x}^2_{n'm'}\right]$$

$$\hat{\text{COV}}(\bar{r}_{nm}{}'\bar{x}n'm')=\frac{1}{n'(n-1)}\left[\sum_{i}^{n}r^2_{im}\,\bar{x}_{im}-n\bar{r}_{nm}\bar{x}_{nm}\right] +\frac{m}{m-1}\left(\frac{1}{n'm'}-\frac{1}{n'm}\right)\left[\bar{y}_{nm}-\frac{1}{n}\sum_{i}^{n}\bar{r}_{im}\,\bar{x}_{im}\right]$$

$$\hat{V}(\bar{y}_{nm})=\frac{1}{n(n-1)}\left[\sum_{i}^{n}\bar{y}^2_{im}-n\bar{y}^2_{nm}\right]$$

$$\hat{V}(\bar{x}_{nm})=\frac{1}{n(n-1)}\left[\sum_{i}^{n}\bar{x}^2_{im}-n\,\bar{x}^2_{nm}\right]$$

$$\hat{\text{COV}}(\bar{y}_{nm},\,\bar{x}_{nm})=\frac{1}{n(n-1)}\left[\sum_{i}^{n}\bar{x}_{im}\bar{y}_{im}-n\bar{x}_{nm}\bar{y}_{nm}\right]$$

and

$$\hat{\text{COV}}(\bar{y}_{nm},\,\bar{x}_{n'm'})=\frac{1}{n'(n-1)}\left[\sum_{i}^{n'}\bar{x}_{im}\bar{y}_{im}-n\bar{x}_{nm}\bar{y}_{nm}\right]$$

$$+\frac{m}{m-1}\left(\frac{1}{n'm'}-\frac{1}{n'm}\right)\left[\frac{1}{nm}\sum_{i}^{n}\sum_{j}^{m}x_{ij}y_{ij}-\frac{1}{n}\sum_{i}^{n}\bar{x}_{im}\bar{y}_{im}\right].$$

5. Numerical Illustration

The results obtained in sections 2 and 4 will now be illustrated with reference to the data on wheat crop, collected from Meerut district of Uttar Pradesh during Rabi season 1965-66 in a survey conducted by the Institute of Agricultural Research Statistics for evolving suitable sampling procedure for obtaining reliable estimates of yield rates of principal cereal crops at Community Development Block-Level.

The design of the survey was one of two phase, two-stage random sampling where the first-stage units were villages and second-stage units were fields. In the first phase of sampling about 50 villages were selected at random in each of the Community Development Blocks (consisting of about 100 villages) in the district and in each of the selected villages, 4 fields were selected at random for obtaining the pre-harvest estimates of yield through eye-estimation of the yield of fields. In the second phase of sampling, a sub-sample of about 25 villages were selected from the first-phase sample and in each of these villages, a random sub-sample of 2 fields was selected from the first-phase sample of 4 fields. For the fields selected in the second phase, the yield of the crop was estimated by crop-cutting experiments.

The data collected in the survey have been utilized to estimate the yield of wheat crop in kilogram per hectare for each block in the district. The results obtained are presented in the following Table. The estimates obtained are presented along with their percentage standard arrors and have been compared with simple estimates based upon the crop-cutting experiments alone. From the Table it is seen that the percentage standard error of T_2 is smaller than the percentage standard error of T_1 in every block of the district.

If no use is made of supplementary information, the estimate of the population mean is simply given by $\bar{y}_{nm}$. This is tabulated in column 4 of the Table. From the comparison of the percentage standard errors of T_2 and $\bar{y}_{nm}$, it is seen that the unbiased estimate obtained from the double sample (T_2) is always superior to the estimate based on the crop-cutting alone in this case.

Blockwise estimates of average dry yield of wheat (kg/ha)

State : *U.P.*, *District* : *Meerut*, *Crop* : *Wheat*, *Year and Season* : *1965-66* (*Rabi*)

Block	*No. of villages selected in the second phase* n	*No. of villages selected in the first phase* n'	*Mean of crop cutting yield* $\bar{y}_{nm}$	*%SE* ($\bar{y}_{nm}$)	*Estimated yield (biased)* T_1	*%SE* (T_1)	*Estimated yield (unbiased)* T_2	*%SE* T_2
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1. Dhaulana	40	180	1011	10.07	947.87	6.86	948.51	5.79
2. Binauli	50	200	1111	8.07	1503.45	7.72	1210.94	5.47
3. Baraut	22	96	1196	11.93	1283.28	11.27	1201.22	10.57
4. Machhra	33	136	1046	11.38	1128.75	10.13	1076.73	8.43
5. Jani Khurd	39	172	1003	9.34	1173.25	9.04	1037.04	8.73
6. Rajpur	38	172	1133	8.53	1095.75	8.14	1046.34	7.98

REFERENCES

1. Goodman, L.A., and Hartley, H.O. (1958) : "The Precision of Unbiased Ratio-Type Estimators", *Journal of the American Statistical Association*, **53** : 491-508.
2. Hartley, H.O., and Ross A. (1954) : "Unbiased Ratio Estimators", *Nature*, **174**: 270.
3. Panse, V.G., Rajagopalan, M. and Pillai, S.S. (1966) : "Estimation of Crop yields for small areas" Biometrics, **22** : 374-84.
4. Robson, D.S. (1957) : Application of Multi-Variate Polykays to the Theory of Unbiased Ratio-Type Estimation, *Journal of the American Statistical Association*, **52** : 511-22.
5. Ross, A. (1960) : Ph. D. thesis, Iowa State University.
6. Singh, D. (1968) : "Contribution to Double Sampling" contributions in Statistics and Agricultural Sciences, Indian Society of Agricultural Statistics.
7. Sukhatme, B.V. (1962) : "Some Ratio-Type Estimators in Two-Phase Sampling". *Journal of the American Statistical Association* **57** : 628-32.
8. Tukey, J.W. (1956) : "Keeping Moment-like Sampling Computations Simple", *Annals of Mathematical Statistics*, **27** : 37-54.